Or

Show that : $\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx = \frac{\pi}{8} \log 2.$

5. Evaluate the following integral by changing the order of integration $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dxdy.$

Or

Evaluate $\int_0^1 \int_0^{1-x^2} \int_0^{x+y^2} x.dxdydz.$

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Roll No. ....

Total No. of Sections : 3

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Annual Examination, 2020

B.C.A. Part II

THEORETICAL FOUNDATION OF COMPUTER SCIENCE

Paper II

[Differentiation and Integration]

Time : Three Hours] [Maximum Marks : 50

Note : *Section 'A', containing 10 very short answer type questions, is compulsory. Section 'B' consists of short answer type questions and Section 'C' consists of long answer type questions. Section 'A' has to be solved first.*

Section 'A'

Answer the following very short answer type questions in one or two sentences. **1×10=10**

1. Write the statement of Rolle's theorem.
2. Expand by Maclaurin's theorem : $y = e^x$.
3. Write Cartesian formula for radius of curvature.
4. Find the asymptotes of curve : $x(x^2 + y^2) = 2ay^2$.

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5. Write the formula for $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

6. Find $\frac{\partial^2 f}{\partial x \partial y}$ if $f = x^3y^2 - xy^5$.

7. Evaluate : $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

8. Evaluate : $\int \frac{dx}{x^2 + a^2}$.

9. Evaluate : $\int_0^1 \int_0^2 (x + y)\, dxdy$.

10. Write the formula for length of curve of the Cartesian equation $y = f(x)$.

Section 'B'

Answer the following short answer type questions with word limit 150–200. **3×5=15**

1. If $f(x) = \frac{x}{(x-1)(x-2)}$, find $f^n(x)$.

Or

Verify Cauchy's mean value theorem for function $f(x) = x^2$ and $g(x) = x^3$ in [1, 2].

2. Trace the curve $r = a(1 + \cos\theta)$.

Or

Prove that for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\rho = \frac{a^2b^2}{p^3}$, p being the length of the perpendicular from the centre (0, 0) upon the tangent at the point (x, y).

3. If $u = f(r)$, where $r^2 = x^2 + y^2$, show that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$

Or

Show that the functions :

$$u = x + y + z,$$

$$v = x - y + z,$$

and $$w = x^2 + y^2 + z^2 - 2yz$$

are not independent to each other. Hence show that

$$(u + v)^2 + (u - v)^2 = 4w.$$

4. Evaluate : $\int \frac{dx}{3 - 2\sin x}$.

5. Prove that the whole length of the curve $x = a\cos^3 t$, $y = a\sin^3 t$ is $6a$.

Or

Evaluate $\iint_R xy\, dxdy$ over the region in the positive quadrant for which $x + y \leq 1$.

Section 'C'

Answer the following long answer type questions with word limit 300–350. **5×5=25**

1. If $y = [x + \sqrt{1+x^2}]^m$, then prove that :

   (i) $(1 + x^2)y_2 + xy_1 - m^2y = 0$.

   (ii) $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + m^2)y_n = 0$.

Or

Use Taylor's theorem prove that :

$$\tan^{-1}(x + h) = \tan^{-1} x + h \sin z. \frac{\sin z}{1} - (h \sin z)^2 \frac{\sin 2z}{2} + (h \sin z)^3 \frac{\sin 3z}{3} - \ldots\ldots + (-1)^{n-1}(h \sin z)^n \frac{\sin hz}{n} + \ldots\ldots$$

where $z = \cot^{-1} x$.

2. If $(-1, 2)$ be point of inflexion of the curve $f(x) = ax^3 + bx^2$, then evaluate a and b.

Or

Find the radius of curvature at any point (r, θ) of the cardioid $r = a(1 + \cos\theta)$.

3. Find the directional derivative of the scalar function $f(x, y, z) = x^2 + xy + z^2$ at the point $A(1, -1, -1)$ in the direction of the line AB where B has co-ordinates $(3, 2, 1)$.

Or

If $x = r\sin\theta\cos\phi$, $y = r\sin\theta.\sin\phi$, $z = r\cos\theta$, then show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}. = r^2\sin\theta$.

4. Evaluate $\int \frac{x.\tan^{-1} x}{(1+x^2)^{3/2}}\, dx$.

Or

Evaluate $\int_0^{\pi/4} \frac{\sin 2\theta}{\sin^4\theta + \cos^4\theta}\, d\theta$.