ZàÎÂà-4. ây÷ §ýLâk¥ (Prove that) :

OR

uâÀ P_n iàm §ýà ¥§ý vkàÂ»þ rÑqÀ Ñ, mà âÂàÈÂàâvâhm yro §ýà ây÷ §ýLâk¥ ß

If is a Legendre's polynomial of degree , then prove that relation :

ZàÎÂà-5. qâÊyãtà Zààmroà $y(0)=0,\ y(\pi)=1$ mnà ÍuwÊào §ý ¡oâÂà

ÄývÂà§ý $I=\int_0^{\pi}\left[(y')^2-y^2\right]dx$ §ýà jÊt tàÂà Öààm §ýLâk¥ ñ

Find the extremal of the functional $I=\int_0^{\pi}\left[(y')^2-y^2\right]dx$ under the conditions $y(0)=0,\ y(\pi)=1$ and subject to constraint

OR

Àâ i wÙà $4x^2+9y^2=36$ ¥w ârÂÀ §ý tÁu v i Ùàt ÀÊã Öààm §ýLâk¥ ñ

Find the shortest distance between the ellipse $4x^2+9y^2=36$ and the point .

----x----

Roll No......................

Total No. of Section : 03

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Code No. : B-241(B)

Annual Examination - 2017

B.Sc.-II

MATHEMATICS

Paper - II

DIFFERENTIAL EQUATIONS

Max.Marks : 50

Time : 3 Hrs. **Min Marks : 17**

¹âq ß h½»' ¡' tÀy ¡âmv i ÙàÊã ZàÎÂà Ñ, âkÂÑ Ñv §ýÊÂàà ¡âÂwàu Ñ ñ h½»' 'r' t v i ÙàÊã ZàÎÂà h½»' 'y' t Àã i £ÙàÊãu ZàÎÂà Ñ ñ h½»' ¡' §ýà yry qÑv Ñv §ýÊ ñ

Note : Section 'A' containing 10 very short questions is compulsory. Section 'B' consists of short answer type questions and Section 'C' consists of long answer type questions. Section 'A' has to be solved first.

$(2n+1)xP_n-nP_{n-1}$ [illegible] $\dfrac{\quad}{\sqrt{a^2+b^2}}$

h½»-'¡' (Section-'A')

âÂàÈÂààâ§ým ¡âm v i £ÙàÊãu ZàÎÂàà §ý £ÙàÊ Àââk¥ ñ (Answer the following very short-answer-type questions.) (1x10=10)

ZàÎÂà-1. ÄývÂà $F(t)=1$ §ýà vàÅvày Ûýqàm Ê Öààm §ýLâk¥ ñ

Find the Laplace transform of the function .

ZàÎÂà-2. ÄývÂà §ýà vàÅvày Ûýqàm Ê Öààm §ýLâk¥ ñ

Find the Laplace transform of the function $F(t)=\sin t\ \cos t$.

ZàÎÂà-3. âÂàÈÂà t y a ¡âÊ §ýà âwvàqÂà §ýÊ§ý ¡ààÎâ§ý ¡w§ýv ytã§ýÊ½à §ýà âÂàtà½à §ýLâk¥ ß

P.T.O.

Find the partial different equation by the elimination of a and b from :

$z = ax + by + ab$

ZàÎÀà-4. ytãSýÊ½à §ýà q½à ytàSýv Öààm §ýLâk¥ ñ

Find the complete integral of the equation $p^2 + q^2 = m^2$.

ZàÎÀà-5. âÂàÈÀààâ§ým ¡wSýv ytãSýÊ½à §ýà wªàJSýÊ½à §ýLâk¥ ß

Classify the following differential equation :

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\,\partial y} + 3\frac{\partial^2 z}{\partial y^2} = 0$$

ZàÎÀà-6. Ñv §ýLâk¥ (Solve) ß

$$r = a^2 t$$

ZàÎÀà-7. J_n ¡àÊ §ýà Êàyâ§ýuÂà yØà âvâh¥ ñ

To write the Wronskian formula of and .

ZàÎÀà-8. vkàÂ»¦ §ýà ¡wSýv ytãSýÊ½à Öààm §ýLâk¥ ñ

Find out Legendre's differential equation.

ZàÎÀà-9. uâÀ wªà qÊ ¥§ý ÄývÂà§ý $I\left[y(x)\right] = \int_0^1 y(x)\,dx$ qáÊSàâxm Ñ¦ mà¦,

$I[1]$ §ýà tàÂà Öààm §ýLâk¥ ñ

If a functional is defined on the class $C[0,1]$,

then find the value of $I[1]$.

ZàÎÀà-10. §ýÂôâu Õà¤à §ýà¦ qáÊSàâxm §ýLâk¥ ñ

Define central field.

h½»¦-'y' (Section-'C')

âÂàÈÀàà¤à§ým Àã i¦ £ÙàÊâu ZàÎÀàà¦ §ý £ÙàÊ Àââk¥ ñ (Answer the following long-answer type questions.) **(5x5=25)**

ZàÎÀà-1. §ýà tàÂà Öààm §ýLâk¥ ñ

Find the value of $L\{\sin at - at\cos at\}$.

OR

ytãSýv ytãSýÊ½à $y(t) = t^2 + \int_0^t y(u)\sin(t-u)\,du$ §ýà Ñv §ýLâk¥ ñ

Solve the integral equation $y(t) = t^2 + \int_0^t y(u)\sin(t-u)\,du$.

ZàÎÀà-2. Ñv §ýLâk¥ (Solve) ß

$[y(x)] = \int_0^{21} y(x)\,dx$ $p + q = x + y + z$

OR

jàÊâq¹¦ âwâo y¡ Ñv §ýLâk¥ (Solve by Charpit's method) ß

ZàÎÀà-3. Ñv §ýLâk¥ (Solve) ß

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$

OR

Ñv §ýLâk¥ (Solve) ß

$$x^2\,\frac{\partial^2 z}{\partial x^2} - y^2\,\frac{\partial^2 z}{\partial y^2} = xy$$

h½»|-'r' (Section-'B')

âÀàÈÂàà¦â§ým vi¦ £ÙàÊãu ZàÎÂàà¦ §ý £ÙàÊ Àããk¥ ñ (Answer the following short-answer type questions.) **(3x5=15)**

ZàÎÂà-1. ÄývÂà $F(t)$ §ýà vàÅvày ÛýqàmÊ Öààm §ýLâk¥, kÑà¦

Find the Lapalce transform of the function $F(t)$, where

OR

ÀÎàà¢u¦ â§ý $L\{t^n e^{at}\} = \frac{\lfloor n}{(p-a)n+1}$; $p > a$.

Show that $L\{t^n e^{at}\} = \frac{\lfloor n}{(p-a)n+1}$; $p > a$.

ZàÎÂà-2. Ñv §ýLâk¥ (Solve) ß

$$x z p + y z q = xy$$

OR

q½à¦ Ñv Öààm §ýLâk¥ (Find the complete integral) ß

ZàÎÂà-3. Ñv §ýLâk¥ (Solve) ß

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h½»|-'r' (Section-'B')

âÀàÈÂàà¦â§ým vi¦ £ÙàÊãu ZàÎÂàà¦ §ý £ÙàÊ Àããk¥ ñ (Answer the following short-answer type questions.) (3x5=15)

ZàÎÂà-1. ÄývÂà $F(t)$ §ýà vàÅvày ÛýqàmÊ Öààm §ýLâk¥, kÑà¦

Find the Lapalce transform of the function $F(t)$, where

OR

ÀÎàà¢u¦ â§ý $L\{t^n e^{at}\} = \frac{\lfloor n}{(p-a)n+1}$; $p > a$.

$pq = xy$

$F(t) = \begin{cases} 4, & 0 < t < 1 \\ 3, & t > 1 \end{cases}$

$\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2} = x^2$

Show that $L\{t^n e^{at}\} = \frac{\lfloor n}{(p-a)n+1}$; $p > a$.

ZàÎÂà-2. Ñv §ýLâk¥ (Solve) ß

$$x z p + y z q = xy$$

OR

q½à¦ Ñv Öààm §ýLâk¥ (Find the complete integral) ß

ZàÎÂà-3. Ñv §ýLâk¥ (Solve) ß

P.T.O.

OR

Ñv §ýLâk¥ (Solve) ß

$$\left(D^2-6DD'+9D'^2\right)z=6x+2y$$

ZàÎÀà-4. ây÷ §ýLâk¥ (Prove that) ß

$$\frac{d}{dx}\left(x^n J_n(x)\right)=x^n J_{n-1}(x)$$

OR

ây÷ §ýLâk¥ (Prove that) ß

$$P_n(-x)=(-1)^n P_n(x)$$

ZàÎÀà-5. ¡ÂmÊàv [0,1] t¦ wSýà ¥w/ §ý rãj §ýL ÀÊã Öààm §ýLâk¥ ñ

Find the distance between the curves and in the interval of .

OR

Äýv Âà§ý , $y(1)=1$, $y(e)=1$ §ý jÊt tàÀà §ýà qÊâÕà½à §ýLâk¥ ñ

Test for the extremum of the functional , $y(1)=1$, $y(e)=1$.

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OR

Ñv §ýLâk¥ (Solve) ß

ZàÎÀà-4. ây÷ §ýLâk¥ (Prove that) ß

$$\frac{d}{dx}\left(x^n J_n(x)\right)=x^n J_{n-1}(x)$$

OR

ây÷ §ýLâk¥ (Prove that) ß

$$P_n(-x)=(-1)^n P_n(x)$$

ZàÎÀà-5. ¡ÂmÊàv [0,1] t¦ wSýà ¥w/ §ý rãj §ýL ÀÊã Öààm §ýLâk¥ ñ

Find the distance between the curves and in the interval of .

OR

Äýv Âà§ý , $y(1)=1$, $y(e)=1$ §ý jÊt tàÀà §ýà qÊâÕà½à §ýLâk¥ ñ

Test for the extremum of the functional , $y(1)=1$, $y(e)=1$.