In the curve $y = c\cos h\left(\frac{x}{c}\right)$, prove that the co-ordinates of the centre of curvature are given by $\alpha = x - y\sqrt{\left(\frac{y^2}{c^2} - 1\right)}$, $\beta = 2y$.

ZàÎÀà-3. $\int \left[\sqrt{\tan x} + \sqrt{\cot x}\right] dx$ §ýà tàÂà Öààm §ýLâk¥ ñ

Find the value of $\int \left[\sqrt{\tan x} + \sqrt{\cot x}\right] dx$.

OR

qÊwvuà¦ $y^2 = 4ax$ ¡àÊ $x^2 = 4ay$ §ý rãj £suâÀà™¦ ÕàØàÄýv §ýà Öààm §ýLâk¥ ñ

Find the area enclosed by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

ZàÎÀà-4. Ñv §ýLâk¥ (Solve) ß

$$\left(D^2 - 4D + 4\right) y = e^x + x^2 + \cos 2x$$

OR

Ñv §ýLâk¥ (Solve) ß

$$xdy - ydx = \sqrt{x^2 + y^2}\, dx$$

ZàÎÀà-5. Zààjv âwjÊ½à §ýL âwâo §ýà ZàuàªÀà §ýÊ§ý Ñv §ýLâk¥ ß

Apply the method of variation of parameters to solve :

$$\left(D^2 + 1\right) y = x$$

OR

âÂàÈÂà uªàqm ytã§ýÊ½àà¦ §ýà Ñv §ýLâk¥ ß

Solve the following simultaneous differential equations :

$$\frac{dx}{dt} = x - 2y, \frac{dy}{dt} = 5x + 3y$$

----x----

Code No. : B-209(B)

Annual Examination - 2017

B.Sc.-I

MATHEMATICS

Paper - II

CALCULUS

Max.Marks : 50

Time : 3 Hrs. **Min Marks : 17**

¹¡ãq ß h½»¦'¡' t¦ Ày ¡âmvi¦ÙàÊã ZàÎÀà Ñ¦, âkÂÑ¦ Ñv §ýÊÂàà ¡âÀàwàu¦ Ñ¦ ñ h½»¦'r' t¦ vi¦ÙàÊã ZàÎÀà h½»¦'y' t¦ Àãi¦ £¦ÙàÊãu ZàÎÀà Ñ¦ ñ h½»¦'¡' §ýà yry¦ qÑv¦ Ñv §ýÊ¦ ñ

Note: Section 'A' , containing 10 very short answer type questions, is compulsory. Section 'B' consists of short answer type questions and Section 'C' consists of long answer type questions. Section 'A' has to be solved first.

h½»¦-'¡'(Section-'A')

âÂàÈÂàà¦â§ým ¡âm vi¦£¦ÙàÊãu ZàÎÀà¦ §ý £¦ÙàÊ Àãâk¥ ñ (Answer the following very short-answer-type questions) (1x10=10)

ZàÎÀà-1. Äýv¦Àà $f(x) = x\sin\frac{1}{x}$ §ý âv¥ $f(0+0)$ Öààm §ýLâk¥ ñ

For the function $f(x) = x\sin\frac{1}{x}$, find $f(0+0)$

ZàÎÀà-2. Äýv¦Àà $\tan^{-1} x$ §ýà ZàyàÊ âvâh¥ ñ

Write the expansion of function $\tan^{-1} x$.

ZàÎÀà-3. ÀÎà¦à¢u¦ â§ý w§ý $y = e^x$ yw¦Øà £qáÊt¦hã ¡wmv Ñ¦ ñ

Show that the curve $y = e^x$ is concave upwards every where.

P.T.O.

ZàÎÀà-4. o¦wàÂmÊ Ê¦hà qÊ vÈr wS¶ýmà kãwà S¶ýà y¦Øà âvàh¥ ñ

Write the formula of chord of curvature perpendicular to the radius vector.

ZàÎÀà-5. $\int e^x \cos h\, x dx$ S¶ýà tàÂà Öààm S¶ýLâk¥ ñ

Find the value of $\int e^x \cos h\, x dx$.

ZàÎÀà-6. $\int_0^{\pi/2} \sin^4 x \cos^2 x dx$ S¶ýà tàÂà Öààm S¶ýLâk¥ ñ

Find the value of $\int_0^{\pi/2} \sin^4 x \cos^2 x dx$.

ZàÎÀà-7. ¡wS¶ýv yt㶶ýÊ½à $\frac{d^2x}{dy^2} + \sqrt{\left[1+\left(\frac{dx}{dy}\right)^2\right]} = 0$ S¶ýL S¶ýàâ¹¦ ¡àÊ iàm Öààm S¶ýLâk¥ ñ

Find the order and degree of the differential equation

$$\frac{d^2x}{dy^2} + \sqrt{\left[1+\left(\frac{dx}{dy}\right)^2\right]} = 0 .$$

ZàÎÀà-8. ¡wS¶ýv yt㶶ýÊ½à $\frac{dy}{dx} = \frac{-\cos y}{y^2 - x\sin y}$ S¶ý unàwm ÑàÂà¡ S¶ýà qÊãÕà½à S¶ýLâk¥ ñ

Examine the differential equation $\frac{dy}{dx} = \frac{-\cos y}{y^2 - x\sin y}$ for exactness.

ZàÎÀà-9. ¡wS¶ýv yt㶶ýÊ½à $x^2\frac{d^2y}{dx^2} - 2\left(x^2+x\right)\frac{dy}{dx} + \left(x^2+2x+2\right)y = 0$ S¶ý âv¥ P, Q, R S¶ýà tàÂà Öààm S¶ýLâk¥ ñ

Find the value of P, Q, R of the differential equation

$$x^2\frac{d^2y}{dx^2} - 2\left(x^2+x\right)\frac{dy}{dx} + \left(x^2+2x+2\right)y = 0 .$$

OR

Ñv S¶ýLâk¥ (Solve) ß $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$

h½»¦-'y' (Section-'C')

âÂàÈÀàà¦â§ým Àã¡¦ £ÙàÊãu ZàÎÀàà¦ S¶ý £ÙàÊ ÀãâÀk¥ ñ (Answer the following long-answer type questions) **(5x5=25)**

ZàÎÀà-1. ¹¦vÊ S¶ý Zàtu y¦ $\log_e \sin x$ S¶ýà $(x-2)$ S¶ýL iàmà¦ t¦ ZàyàÊ S¶ýLâk¥ ñ

Expand $\log_e \sin x$ in powers of $(x-2)$ by Taylor's theorem.

OR

uâÀ $y = \left(\sin^{-1} x\right)^2$, mà¦ ây÷ S¶ýLâk¥ â§ý $\left(1-x^2\right)y_2 - xy_1 = 2$ mnà $\left(1-x^2\right)y_{n+2} - (2n+1)x\,y_{n+1} - n^2 y_n = 0$.

If $y = \left(\sin^{-1} x\right)^2$, then prove that $\left(1-x^2\right)y_2 - xy_1 = 2$ and $\left(1-x^2\right)y_{n+2} - (2n+1)x\,y_{n+1} - n^2 y_n = 0$.

ZàÎÀà-2. wS¶ý $x^3 - 2y^3 + 2x^2y - xy^2 + xy - y + 1 = 0$ S¶ýL ¡ÂàmÐqâÎà¦uà¦ Öààm S¶ýLâk¥ ñ

Find the asymptotes of the curve

$x^3 - 2y^3 + 2x^2y - xy^2 + xy - y + 1 = 0$

OR

ây÷ S¶ýLâk¥ â§ý wS¶ý $y = c\cos h\left(\frac{x}{c}\right)$ S¶ý âv¥ wS¶ýmà S¶ýÂô S¶ý âÂàÀ¦Î¦à¦S¶ý $\alpha = x - y\sqrt{\left(\frac{y^2}{c^2} - 1\right)}$, $\beta = 2y$ Ñ¦ ñ

P.T.O.

ZàÎÂà-10. ytãSýÊ½à $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$ §ýL ³uàâtmãu Íuà©uà âvâh¥ ñ

Write geometrical interpretation of the differential equation $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$.

h½»-'r'(Section-'B')

âÂàÈÂàà¹â§ým vi£ÙàÊãu ZàÎÂàà¹ §ý£ÙàÊ ÀãâK¥ ñ (Answer the following short-answer type questions) (3x5=15)

ZàÎÂà-1. $\in-\delta$ m§ýÂàâSý Sý Zàuà¢a y¡ y¼uàâqm §ýLâK¥ : $\lim_{x\to 1}(2x+7)=9$

Using $\in-\delta$ technique, verify that $\lim_{x\to 1}(2x+7)=9$.

OR

ây÷ §ýLâK¥ âSý $x=2$ qÊ ÄývÂà $f(x)=3x^2+2x+1$ ymm¡ Ñ¡ ñ

Prove that the following function is continuous at $x=2$, $f(x)=3x^2+2x+1$

ZàÎÂà-2. úÀuàs $r=a(1+\cos\theta)$ §ý âSýyã ârÀÀ¡ (r,θ) qÊ wSýmà âØà³uà Öààm §ýLâK¥ ñ

Find the radius of curvature at any point (r,θ) of the cardioid $r=a(1+\cos\theta)$.

OR

wSý $y^2(2a-x)=x^3$ Sýà ¡ÂàÊh½à §ýLâK¥ ñ

Trace the curve $y^2(2a-x)=x^3$.

P.T.O.

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ZàÎÂà-10. ytãSýÊ½à $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$ §ýL ³uàâtmãu Íuà©uà âvâh¥ ñ

Write geometrical interpretation of the differential equation $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$.

h½»-'r'(Section-'B')

âÂàÈÂàà¹â§ým vi£ÙàÊãu ZàÎÂàà¹ §ý£ÙàÊ ÀãâK¥ ñ (Answer the following short-answer type questions) (3x5=15)

ZàÎÂà-1. $\in-\delta$ m§ýÂàâSý Sý Zàuà¢a y¡ y¼uàâqm §ýLâK¥ : $\lim_{x\to 1}(2x+7)=9$

Using $\in-\delta$ technique, verify that $\lim_{x\to 1}(2x+7)=9$.

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ây÷ §ýLâK¥ âSý $x=2$ qÊ ÄývÂà $f(x)=3x^2+2x+1$ ymm¡ Ñ¡ ñ

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Find the radius of curvature at any point (r,θ) of the cardioid $r=a(1+\cos\theta)$.

OR

wSý $y^2(2a-x)=x^3$ Sýà ¡ÂàÊh½à §ýLâK¥ ñ

Trace the curve $y^2(2a-x)=x^3$.

P.T.O.

ZàÎÀà-3. $\int \frac{dx}{4+5\cos x}$ §ýà tàÂà Öààm §ýLâk¥ ñ

Evaluate $\int \frac{dx}{4+5\cos x}$.

OR

ây÷ §ýLâk¥ âS ý $\int_0^1 x^2\left(1-x^2\right)^{3/2}\,dx=\frac{\pi}{32}$

Prove that $\int_0^1 x^2\left(1-x^2\right)^{3/2}\,dx=\frac{\pi}{32}$.

ZàÎÀà-4. wS ý §ýv $r^n\sin n\theta=a^n$ §ýà vrS ýà½àãu y²ÀÃ Öààm §ýLâk¥, kÑà a wS ý §ýv §ýà Zàà j v Ñ ñ

Find the orthogonal trajectories of given family of curves $r^n\sin n\theta=a^n$ a, being parameter of family of curve.

OR

¡ wS ýv ytãS ýÊ½à $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=2\log x$ §ýà Ñv §ýLâk¥ ñ

Solve the differential equation $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=2\log x$.

ZàÎÀà-5. ¡ wS ýv ytãS ýÊ½à $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-3\right)y=e^{x^2}$ §ýà qÊS ý ÄývÂà Öààm §ýLâk¥ ñ

Find the complimentary function of the differential equation $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-3\right)y=e^{x^2}$.

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ZàÎÀà-3. $\int \frac{dx}{4+5\cos x}$ §ýà tàÂà Öààm §ýLâk¥ ñ

Evaluate $\int \frac{dx}{4+5\cos x}$.

OR

ây÷ §ýLâk¥ âS ý $\int_0^1 x^2\left(1-x^2\right)^{3/2}\,dx=\frac{\pi}{32}$

Prove that $\int_0^1 x^2\left(1-x^2\right)^{3/2}\,dx=\frac{\pi}{32}$.

ZàÎÀà-4. wS ý §ýv $r^n\sin n\theta=a^n$ §ýà vrS ýà½àãu y²ÀÃ Öààm §ýLâk¥, kÑà a wS ý §ýv §ýà Zàà j v Ñ ñ

Find the orthogonal trajectories of given family of curves $r^n\sin n\theta=a^n$ a, being parameter of family of curve.

OR

¡ wS ýv ytãS ýÊ½à $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=2\log x$ §ýà Ñv §ýLâk¥ ñ

Solve the differential equation $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=2\log x$.

ZàÎÀà-5. ¡ wS ýv ytãS ýÊ½à $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-3\right)y=e^{x^2}$ §ýà qÊS ý ÄývÂà Öààm §ýLâk¥ ñ

Find the complimentary function of the differential equation $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-3\right)y=e^{x^2}$.