Find the equation of cylinder with generator parallel to $x = -\frac{y}{2} = \frac{z}{3}$ and guiding curve $x^2 + 2y^2 = 1,\ z = 3$.

ZàÎÂà-5. ÀÎàà¢u| â§ý ytmv $x+2y-2z=4$ qÊwvuk $3x^2+4y^2=24z$ §ýà ÐqÎà §ýÊmà Ñ| ¡àÊ ÐqÎà ârÂÀ| Öààm §ýLâk¥ ñ

Show that the plane $x+2y-2z=4$ touches the paraboloid $3x^2+4y^2=24z$. Also find the point of contact.

OR

ytâ§ýÊ½à $2x^2+2y^2+z^2+2yz-2zx-4xy+x+y=0$ §ýà Zàtàâ½à§ý Úq t| ytàÂàuÂà §ýLâk¥ mnà ¢y§ý Îààx| §ýà âÂàÀÎàà§ý ¥w| ¡Õàà| §ý ytâ§ýÊ½à Öààm §ýLâk¥ ñ

Reduce the equation $2x^2+2y^2+z^2+2yz-2zx-4xy+x+y=0$ to the standard form and find the coordinates of its vertex and equation of axes.

----x----

Roll No.....................

Total No. of Section : 03
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Code No. : B-210(B)

Annual Examination - 2017

B.Sc.-I

MATHEMATICS

Paper - III

VECTOR ANALYSIS AND GEOMETRY

Max.Marks : 50
Time : 3 Hrs. **Min Marks : 17**

¹âq ß h½»| '¡' t| Ày ¡âmv i|ÙàÊã ZàÎÂà Ñ|, âkÂÑ| Ñv §ýÊÂàà ¡âÂàwàu| Ñ| ñ h½»| 'r' t| v i|ÙàÊã ZàÎÂà h½»| 'y' t| Àâ i| £ÙàÊâu ZàÎÂà Ñ| ñ h½»| '¡' §ýà yry| qÑv| Ñv §ýÊ| ñ

Note: Section 'A' , containing 10 very short answer type questions, is compulsory. Section 'B' consists of short answer type questions and Section 'C' consists of long answer type questions. Section 'A' has to be solved first.

h½»|-'¡' (Section-'A')

âÂàÈÂàààâ§ým ¡âm vi|£ÙàÊâu ZàÎÂàà| §ý £ÙàÊ ¥§ý uà Àà| qâQýuà| t| À| ñ

(Answer the following very short-answer-type questions in one or two lines.) (1x10=10)

ZàÎÂà-1. yâÀÎà âØà§ý ªà½àÂà §ýà yØà âvâh¥ ñ

Write formula of vector triple product.

ZàÎÂà-2. yâÀÎà âky§ýL âÀÎàà ¡jÊ Ñà| §ýà ¨uà ZààmrÂo Ñ| ?

Write the condition for a vector to be in constant direction.

ZàÎÂà-3. Ð¹|à¨y Zàt|u âvâh¥ ñ

Write Stoke's theorem.

ZàÎÂà-4. ytmv t| ªàãÂà Zàt|u §ýà| âvâh¥ ñ

Write Green's theorem in the plane.

ZàÎÂà-5. âÕ i|àmãu Íuàq§ý ytâ§ýÊ½à §ýr ¥§ý wÙà §ýà| âÂàÚâqm §ýÊªàà ?

When will general equation of second degree represents a circle?

ZàÎÂà-6. £y Îàà§ýw §ýà o|wãu ytâ§ýÊ½à âvâh¥ âky§ýL Âàààs o|w Ñ| mnà ¢y§ýà ¡Õà ZààÊâs§ý Êhà y| α §ýà½à rÂààmà Ñ| ñ

P.T.O.

Write polar equation of conic with focus at pole and axis making an angle α with initial line.

ZâÎÀà-7. ytmv §ýà ytâ§ýÊ½à âvâh¥ kà ¡Õàà y¦ ¡mß h»¦ 2, 3, -4 §ýà¹¦mà Ñ¦ ñ

Write equation of a plane with intercepts 2, 3, -4 on axes.

ZâÎÀà-8. r¦vÂà §ýà ytlà¢¥ ñ

Explain the cylinder.

ZâÎÀà-9. Àãi¦wÙàâu qÊwvuk §ýà ytâ§ýÊ½à âvâh¥ ñ

Write the equation of Elliptic paraboloid.

ZâÎÀà-10. ¥§ý Àãi¦wÙàk §ý yÂàààs Îàà§ýwk §ýà yØà âvâh¥ ñ

Write formula of confocal conicoids of a ellipsoid.

h½»¦-'r' (Section-'B')

âÂàÈÂààà§ým ZâÎÂàà¦ §ý £ÙàÊ Àãâk¥ñ (Answer the following questions.

(3x5=15)

ZâÎÀà-1. uâÀ $\bar{a}, \bar{b}, \bar{c}$ mâÂà ¡ytmvâu yâÀÎà Ñ¦ mà¦ ÀÎàà¢u¦ â§ý

$$\left[\bar{a} \times \bar{b},\ \bar{b} \times \bar{c},\ \bar{c} \times \bar{a}\right] = \left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$$

For three non-coplanar vectors $\bar{a}, \bar{b}, \bar{c}$ show that

$$\left[\bar{a} \times \bar{b},\ \bar{b} \times \bar{c},\ \bar{c} \times \bar{a}\right] = \left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$$

OR

uâÀ $\bar{f} = xy^2 i + 2x^2 yzj - 3yz^2 k$, mr ârÂÀ¦ $(1, -1, 1)$ qÊ Öààm §ýLâk¥ ñ

i) $div\,\bar{f}$ ii) $curl\,\bar{f}$

If $\bar{f} = xy^2 i + 2x^2 yzj - 3yz^2 k$, then find i) $div\,\bar{f}$ ii) $curl\,\bar{f}$ on point $(1, -1, 1)$.

ZâÎÀà-2. $\int_C \bar{F}.d\bar{r}$ §ýà t¦Ìuà¦§ýÂà §ýLâk¥, kÑà¦ $\bar{F} = x^2 y^2 i + y\, j$ ¡à¦Ê $C, y^2 = 4x$, xy ytmv t¦ $(0,0)$ y¦ $(4,4)$ m§ý Ñ¦ ñ

Evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = x^2 y^2 i + y\, j$ and C is $y^2 = 4x$, in xy plane from $(0,0)$ to $(4,4)$.

ZâÎÀà-2. Ð¹¦à¦¨y Zàtu §ýà y¼uàqÂà §ýLâk¥, kr $\bar{F} = yi + zj + xk$ mnà q¦™¦ S ªààv¦ $x^2 + y^2 + z^2 = 1$ §ýà xy ytmv §ý ¤ýqÊ §ýà sàªà Ñ¦ ñ

Verify Stoke's theorem for $\bar{F} = yi + zj + xk$ and surface S is upper part of xy plane of the sphere $x^2 + y^2 + z^2 = 1$.

OR

t¦Ìuà¦§ýÂà §ýLâk¥ $\iint_S \bar{F}.\hat{n}\, dS$ kÑà¦ $\bar{F} = 4zxi - y^2 j + yzk$ mnà $S, x = 0,\ x = a,\ y = a,\ y = 0,\ z = 0,\ z = a$ y¦ qáÊr÷ iÀà §ýà q¦™¦ Ñ¦ ñ

Evaluate $\iint_S \bar{F}.\hat{n}\, dS$ where $\bar{F} = 4zxi - y^2 j + yzk$ and S is a cube bounded by $x = a,\ x = 0,\ y = a,\ y = 0,\ z = 0,\ z = a$.

ZâÎÀà-3. â§ýyã Îàà§ýw t¦ ây÷ §ýLâk¥ â§ý vÈrÚq ÂàààsªÀm kãwà¡à¦ §ý Íu¼§ýtà¦ §ýà uàªà ¡jÊ Ñàmà Ñ¦ ñ

In a conic, prove that the sum of reciprocal of perpendicular focal chord is constant.

OR

Îàà§ýw $21x^2 - 6xy + 29y^2 + 6x - 58y - 151 = 0$ §ýà ¡ÂàÊh½à §ýLâk¥ ñ

Trace the conic $21x^2 - 6xy + 29y^2 + 6x - 58y - 151 = 0$.

ZâÎÀà-4. ây÷ §ýLâk¥ â§ý ytâ§ýÊ½à

$2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$ ¥§ý Îà¦§ý §ýà âÂàÚâqm §ýÊmà Ñ¦ ñ ¢y§ý Îààx¦ §ýà âÂàÀÎàà§ý rmà¢¥ ñ

Prove that the equation

$2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$

represents a cone. Find co-ordinate of its vertex.

OR

£y r¦vÂà §ýà ytâ§ýÊ½à Öààm §ýLâk¥ âky§ý kÂà§ý yÊv ÊhÃ $x = -\frac{y}{2} = \frac{z}{3}$ §ý ytÂàààmÊ Ñ¦ mnà âÂàÀÎà§ý w§ý Àãi¦wÙà $x^2 + 2y^2 = 1,\ z = 3$ Ñ¦ ñ

P.T.O.

OR

ÀÎàà¢u¦ â§ý $\iint_S (axi+byj+czk).ndS = \frac{4}{3}\pi(a+b+c)$ kÑà¦ S ªààv¦ $x^2+y^2+z^2=1$ §ýà yÈq½à¦ q¦™¦ Ñ¦ ñ

Show that $\iint_S (axi+byj+czk).ndS = \frac{4}{3}\pi(a+b+c)$, where S is whole surface of sphere of $x^2+y^2+z^2=1$.

ZàÎÂà-3. ây÷ §ýLâk¥ â§ý ytâ§ýÊ½à $\frac{l}{r}=1-e\cos\theta$ mnà $\frac{l}{r}=-1-e\cos\theta$ ¥§ý Ñã Îàà§ýw §ýà¦ âÂàÚâqm §ýÊm¦ Ñ¦ ñ

Prove that equations $\frac{l}{r}=1-e\cos\theta$ and $\frac{l}{r}=-1-e\cos\theta$ represent the same conic.

OR

Îàà§ýw $x^2+2y^2=2$ y¦ yÂàààs Îàà§ýw §ýà ytâ§ýÊ½à Öààm §ýLâk¥ kà¦ ârÂÀ¦ (1,1) y¦ Ñà§ýÊ kàmà Ñ¦ ñ

Find equation of confocal conic of conic $x^2+2y^2=2$ passes through $(1,1)$.

ZàÎÂà-4. ªààv¦ $x^2+y^2+z^2=a^2$ §ý (α,β,γ) ârÂÀ¦ qÊ ÐqÎà¦ mv §ýà ytâ§ýÊ½à âÂà§ýààv¥ ñ

Find tangent plane of sphere $x^2+y^2+z^2=a^2$ at (α,β,γ).

OR

vÈr wÙààu r¦vÂà §ýà ytâ§ýÊ½à Öààm §ýLâk¥ âky§ýL âØà³uà 2 mnà ¡Õà Ê¦hà $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$ Ñ¦ ñ

Find equation of right circular cylinder with radius 2 and axis on straight line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$.

P.T.O.

(3) **Code No. : B-210(B)**

OR

ÀÎàà¢u¦ â§ý $\iint_S (axi+byj+czk).ndS = \frac{4}{3}\pi(a+b+c)$ kÑà¦ S ªààv¦ $x^2+y^2+z^2=1$ §ýà yÈq½à¦ q¦™¦ Ñ¦ ñ

Show that $\iint_S (axi+byj+czk).ndS = \frac{4}{3}\pi(a+b+c)$, where S is whole surface of sphere of $x^2+y^2+z^2=1$.

ZàÎÂà-3. ây÷ §ýLâk¥ â§ý ytâ§ýÊ½à $\frac{l}{r}=1-e\cos\theta$ mnà $\frac{l}{r}=-1-e\cos\theta$ ¥§ý Ñã Îàà§ýw §ýà¦ âÂàÚâqm §ýÊm¦ Ñ¦ ñ

Prove that equations $\frac{l}{r}=1-e\cos\theta$ and $\frac{l}{r}=-1-e\cos\theta$ represent the same conic.

OR

Îàà§ýw $x^2+2y^2=2$ y¦ yÂàààs Îàà§ýw §ýà ytâ§ýÊ½à Öààm §ýLâk¥ kà¦ ârÂÀ¦ (1,1) y¦ Ñà§ýÊ kàmà Ñ¦ ñ

Find equation of confocal conic of conic $x^2+2y^2=2$ passes through $(1,1)$.

ZàÎÂà-4. ªààv¦ $x^2+y^2+z^2=a^2$ §ý (α,β,γ) ârÂÀ¦ qÊ ÐqÎà¦ mv §ýà ytâ§ýÊ½à âÂà§ýààv¥ ñ

Find tangent plane of sphere $x^2+y^2+z^2=a^2$ at (α,β,γ).

OR

vÈr wÙààu r¦vÂà §ýà ytâ§ýÊ½à Öààm §ýLâk¥ âky§ýL âØà³uà 2 mnà ¡Õà Ê¦hà $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$ Ñ¦ ñ

Find equation of right circular cylinder with radius 2 and axis on straight line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$.

P.T.O.

ZàÎÂà-5. ¡âmqÊwvuk $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$ §ý ârÂÀ $(2, 3, -4)$ y¦ kàÂà wàv¦ kÂà§ýà¦ §ý ytã§ýÊ½à Öàam §ýLâk¥ ñ

Find equation of generators of hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$ which passes through $(2, 3, -4)$.

OR

ây÷ §ýLâk¥ â§ý Îàà¦§ýwk $ax^2+by^2+cz^2=1$ §ýà Îà¦§ý $\frac{x^2}{b+c}+\frac{y^2}{c+a}+\frac{z^2}{a+b}=0$ §ý ÐqÎà¦mv õàÊà Zàâm°²¦¦À ¥§ý yt§ýàâ½à§ý ¡âmqÊwvu Ñ¦ ñ

Prove that the intersection of tangent plane of cone $\frac{x^2}{b+c}+\frac{y^2}{c+a}+\frac{z^2}{a+b}=0$ with conicoid $ax^2+by^2+cz^2=1$ is a rectangular hyperbola.

h½»¦-'y' (Section-'C')

âÂàÈÂàà¦â§ým ZàÎÂàà¦ §ý £ÙàÊ Àãâk¥ ñ (Answer the following questions) **(5x5=25)**

ZàÎÂà-1. uâÀ $\bar{r}$ â§ýyã ârÂÀ¦ §ýà ÒÐnâm yâÀÎà Ñ¦ mnà r £y§ýà tàqà§ý Ñ¦ mà¦ n §ýà tàÂà Öàam §ýLâk¥ kr $(r^n\,\bar{r})$ qâÊÂààv§ýLu Ñ¦ ñ

If $\bar{r}$ is position vector of any point with modulus r then find the value of n when $(r^n\,\bar{r})$ is solenoidal.

OR

ây÷ §ýLâk¥ â§ý $[\bar{a}\ \bar{b}\ \bar{c}]$ §ýà Íu¼§ýt $[\bar{a}'\ \bar{b}'\ \bar{c}']$ Ñ¦ ñ

Prove that reciprocal of $[\bar{a}\ \bar{b}\ \bar{c}]$ is $[\bar{a}'\ \bar{b}'\ \bar{c}']$.

(4) **Code No. : B-210(B)**

ZàÎÂà-5. ¡âmqÊwvuk $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$ §ý ârÂÀ $(2, 3, -4)$ y¦ kàÂà wàv¦ kÂà§ýà¦ §ý ytã§ýÊ½à Öàam §ýLâk¥ ñ

Find equation of generators of hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$ which passes through $(2, 3, -4)$.

OR

ây÷ §ýLâk¥ â§ý Îàà¦§ýwk $ax^2+by^2+cz^2=1$ §ýà Îà¦§ý $\frac{x^2}{b+c}+\frac{y^2}{c+a}+\frac{z^2}{a+b}=0$ §ý ÐqÎà¦mv õàÊà Zàâm°²¦¦À ¥§ý yt§ýàâ½à§ý ¡âmqÊwvu Ñ¦ ñ

Prove that the intersection of tangent plane of cone $\frac{x^2}{b+c}+\frac{y^2}{c+a}+\frac{z^2}{a+b}=0$ with conicoid $ax^2+by^2+cz^2=1$ is a rectangular hyperbola.

h½»¦-'y' (Section-'C')

âÂàÈÂàà¦â§ým ZàÎÂàà¦ §ý £ÙàÊ Àãâk¥ ñ (Answer the following questions) **(5x5=25)**

ZàÎÂà-1. uâÀ $\bar{r}$ â§ýyã ârÂÀ¦ §ýà ÒÐnâm yâÀÎà Ñ¦ mnà r £y§ýà tàqà§ý Ñ¦ mà¦ n §ýà tàÂà Öàam §ýLâk¥ kr $(r^n\,\bar{r})$ qâÊÂààv§ýLu Ñ¦ ñ

If $\bar{r}$ is position vector of any point with modulus r then find the value of n when $(r^n\,\bar{r})$ is solenoidal.

OR

ây÷ §ýLâk¥ â§ý $[\bar{a}\ \bar{b}\ \bar{c}]$ §ýà Íu¼§ýt $[\bar{a}'\ \bar{b}'\ \bar{c}']$ Ñ¦ ñ

Prove that reciprocal of $[\bar{a}\ \bar{b}\ \bar{c}]$ is $[\bar{a}'\ \bar{b}'\ \bar{c}']$.